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Combined estimation method for inertia properties of STSAT-3 †

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Abstract

In general, the information of mass properties is required to control a spacecraft. The mass properties, mass and inertia, are changed by some activities, e.g., consumption of propellant, deployment of solar panel, sloshing, etc. Various estimation methods have been studied to obtain the accurate mass properties. The gyro-based attitude data including noise and bias needs to be compensated for improvement of attitude control accuracy. In this paper, several filtering methods have been investigated for the inertia estimation of STSAT-3 and a new method is suggested for better performance. First, we filter the gyro noise with the extended Kalman filter, and then estimate the inertia using the batch method. The estimated inertia is then fed back to nominal values of the extended Kalman filter. This process is iterated until the estimated properties are converged. The performance of the suggested method has been verified for the case of STSAT-3, Korea Science Technology Satellite.

Keywords: Mass properties; Batch method; Extended Kalman filter; Iteration; STSAT-3

1. Introduction

Before launch, the mass properties of spacecraft should be measured and used for attitude control of the spacecraft. In orbit, the change of the mass properties depends on consumption of propellant or any events which substantially influence the inertia matrix. The variation of the mass properties affects the performance of the attitude control. Accurate information of the properties is required for reliable and efficient attitude control. The least square algorithm is the most popular method for this purpose.

Tanygin and Williams developed the algorithm based on the least squares to estimate the mass properties using coasting maneuver of the vehicle [1]. Bergmann et al. developed the recursively estimating method for the mass properties using the given noisy measurement of the translational and angular velocities of the vehicle and the known control inputs [2]. Keim et al. suggested new formulation for spacecraft inertia estimation from test data using the constrained least squares [3]. Palimaka and Burlton presented the application of a weighted-least-squared estimation method for the determination of a spacecraft's mass properties [4]. Wilson et al. developed the multiple concurrent recursive least squares, which is the on-line mass-property identification using the gyro-based attitude data [5].

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The accuracy of the estimation of the mass properties might be deteriorated when the gyro-based data with noise is directly used. There have been many filters to reduce the effects of the noise, such as the Butterworth, the zero-phase shift, the extended Kalman filter (EKF), etc.

Psiaki et al. applied the Kalman filter for estimating three-axis attitude, attitude rate, and disturbance torque for a gravity-gradient stabilized spacecraft [6]. Herman suggested the techniques for the attitude determination and control of the SUNSAT microsatellite using the Kalman filter to estimate full state attitude in his thesis [7]. Kultu et al. presented the state estimation algorithm for inertia estimation using the EKF [8]. Alminde applied the EKF and the Butterworth filter for estimating the torque of a microsatellite [9]. Yang et al. showed that the EKF was the best method in reducing the noise [10].

We propose a batch least squares method combined with the EKF in this paper. First, the least square algorithm for inertia estimation is reviewed in the next section.

2. Least squares algorithm

The equation of motion for a rigid spacecraft under control input can be written as follows:

$$\begin{aligned} \dot{\omega} &= -I^{-1}\omega^{\times} I\omega + I^{-1}u \\ y &= \omega \end{aligned} \tag{1}$$

where I is the inertia matrix of the spacecraft, ω the angular rates of the spacecraft, u the torque applied to the spacecraft, $\times$ the vector cross product, and y the measured angular rates [11].

The torque u is generated by various causes, such as reaction wheel, magnetic torque bar, disturbances, etc. For applying the batch method, Eq. (1) is reformulated into the regression model as follows:

$$z = Hx \tag{2}$$

where H is the regressor matrix formed by the angular velocities and accelerations, x the parameter vector to be estimated, and z the measurement vector.

Minimize the residual cost function J as follows:

$$J = \frac{1}{2}\left(z - Hx\right)^T \left(z - Hx\right) \tag{3}$$

We obtain the least square estimate $\hat{x}$ as follows:

$$\hat{x} = \left(H^T H\right)^{-1} H^T z \tag{4}$$

Here, the measured data z should be non-zero vector and the regressor matrix H must have the full rank [12].

The information of the angular velocities and accelerations is needed for the regressor matrix in Eqs. (2) and (4). The element of the regressor matrix depends on the gyro-based attitude data, which includes noise and bias. It is quite apparent that the accuracy of the estimation is not guaranteed without noise reduction. Therefore, compensation of the noise is needed to improve the estimation accuracy.

3. Noise reduction methods

Three methods are considered for noise reduction: the Butterworth, the zero-phasing, and the EKF.

The Butterworth filter

The Butterworth filter has a frequency response which is as flat as possible in the passbands. Transfer function of this filter has the form as follows [13]:

$$H(s) = \frac{G_0}{\prod_{k=1}^{n}\left(s - s_k\right)/\omega_c} \tag{5}$$

where G_0 is the static gain, ω_c the cut-off frequency, and s_k the pole which is calculated by the cut-off frequency and the order of the filter. The cut-off frequency is a boundary in a frequency response of the system. We choose some proper cut-off frequencies considering the frequency response of the spacecraft and the order of the filter to reduce the noise.

The zero-phase shift filter

The zero-phase shift filter has the zero-phase change about

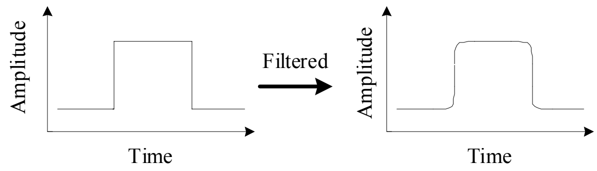

Fig. 1. Concept of the zero-phase shift filter.

the input signal in all of passbands after filtering. This affects the amplitude spectrum without affecting the phase of the input. As shown in Fig. 1, this filter performs both the forward and the reverse directions.

The filtering in the reverse direction is performed after the forward direction. The result has the zero-phase change and the sequence double the filter order [13].

$$Y_O = \left(Y_F + Y_R\right)/2 \tag{6}$$

where Y_O is the output of the zero-phase shift filter, Y_F the output of the low pass filter in forward filtering process, and Y_R the output of the low pass filter in backward filtering process.

The value of the zero-phase shift filter output is determined by taking the average value between the filtering processes at each data point [14].

The dynamic extended Kalman filter

The Kalman filter is one of best methods for state estimation based on minimizing the residual of measurements. The continuous-discrete EKF is applied here. The state vector $\vec{x}$ can be set as follows:

$$\vec{x} = \begin{bmatrix}\omega_1 & \omega_2 & \omega_3\end{bmatrix}^T \tag{7}$$

We use the Eq. (1) as the system equation. The system propagation equations are [15]:

$$\begin{aligned}\dot{\hat{x}}(t) &= f\left(\hat{x},u,t\right) + G(t)w(t)\\ \dot{P} &= FP + PF^T + GQG^T\end{aligned} \tag{8}$$

and the update equations are

$$\begin{aligned}\hat{x}_k^+ &= \hat{x}_k^- + \bar{K}_k\left(z_k - h_k\left(\hat{x}_k^-\right)\right)\\ P_k^+ &= \left(I - \bar{K}_k H_k\right)P_k^-\end{aligned} \tag{9}$$

where

$$\bar{K}_k = P_k^- H_k^T\left[H_k P_k^- H_k^T + R_k\right]^{-1} \tag{10}$$

In Eqs. (8)-(10), subscript k indicates the discrete time step, (−) the predicted state, (+) the estimated state, and Jacobian matrices $F \equiv \partial f/\partial x$, $H \equiv \partial h/\partial x$. $\bar{K}$ is the Kalman gain, P the covariance matrix, and Q, R the state noise and the measurement noise covariance matrix, respectively.

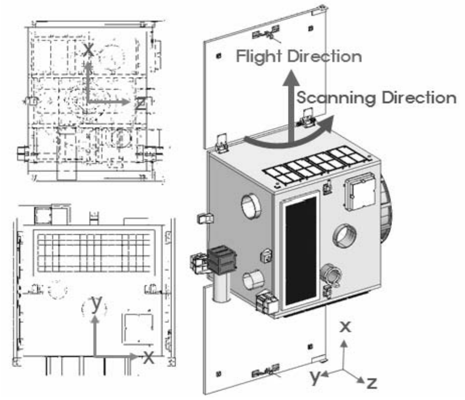

Fig. 2. The reference frame of STSAT-3.

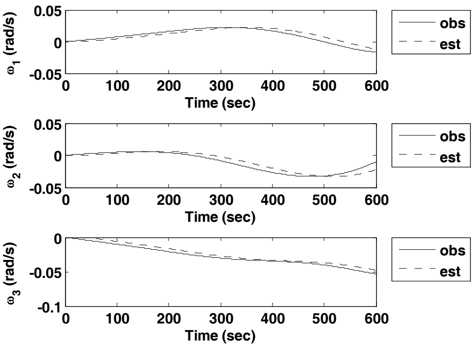

Fig. 3. The angular rates data under the Butterworth filter.

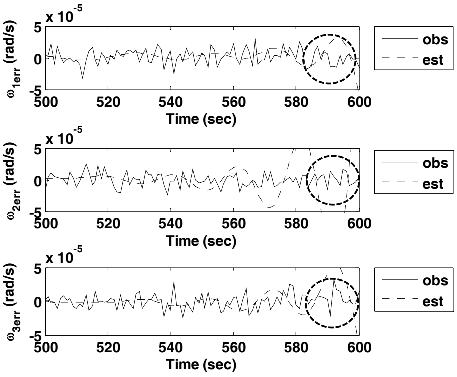

Fig. 4. Error of the angular rates data under the zero-phase shift filter.

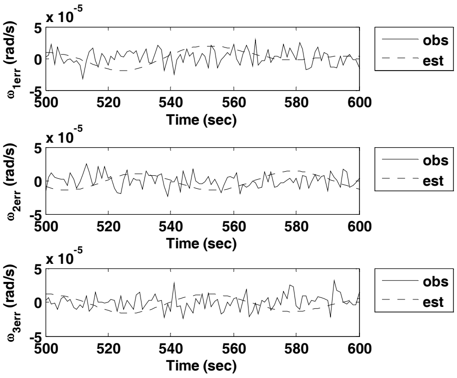

Fig. 5. Error of the angular rates data under the EKF.

Table 1. Noise filtering results.

Filters	RMS value of noise (10^{-4} *rad/s*)		
	ω_1	ω_2	ω_3
None	0.4007	0.4067	0.4016
Butterworth	181.6198 (1.8350)	199.9521 (2.0980)	144.6329 (1.4580)
Zero-phase shift	0.2266	0.7332	0.2945
EKF	0.1550	0.2797	0.2178

Table 2. Least square estimation results.

	Inertia ($kg\text{-}m^2$)		
	I_{xx}	I_{yy}	I_{zz}
True	14.2	17.3	20.3
Butterworth	14.0318	16.9309	19.9163
Zero-phase shift	13.8270	16.4673	v
EKF	14.1834	17.1198	20.1718

The filter application on STSAT-3

The mass properties of the deployed shaped STSAT-3 are estimated under two assumptions. First, the origin of the axis is the center of the contact surface between the adapter of the thruster and the structure of the satellite as shown in Fig. 2. Second, the weight of harness is assumed such that the mass is a uniform distribution about the structure [16].

The input torques and the angular rates are needed for estimation. Four reaction wheels of $5\ mNm$ are used to generate the command torques, and gyroscopes with RMS noise $10^{-5}\ rad/s$ are used to measure the angular rates.

Three filters mentioned above are applied to reduce the noise effect, then the results in comparison with measurements are shown in Figs. 3-5.

As shown in Fig. 4, a phase shift occurs and circles are marked to focus on it. The zero-phase shift filter method is so data-adaptive that the magnitude of phase shift is dependent on the measurement.

The performance comparison of the three filters is listed in Table 1.

The Butterworth filter shows the worst result among the filters due to the phase shift. It may possibly get better performance through elimination of the phase shift manually as shown in parenthesis. The performance of the Butterworth filter depends on the following two important factors; the order of the Butterworth filter and the phase shift. If we increased the order of the Butterworth filter, noise can then be reduced but the phase shift is increased, and vice versa. The zero-phase shift filter is used for eliminating the phase shift. The filter provides better estimation result than the Butterworth filter. However, it can't be used in real-time application. The EKF is a better result than the other two in the aspect of noise reduction. It, however, needs the a priori (nominal) value of the inertia properties.

After the process of filtering, the mass properties are estimated using the batch least square method and the results are

Table 3. Estimation results with different nominal values.

	Inertia (kg-m^2)		
	I_{xx}	I_{yy}	I_{zz}
True	14.2	17.3	20.3
Nominal (10% err.)	14.6438	17.9490	20.9989
Nominal (20% err.)	13.3134	15.7159	18.7837
Nominal (30% err.)	12.2545	14.4039	17.5486

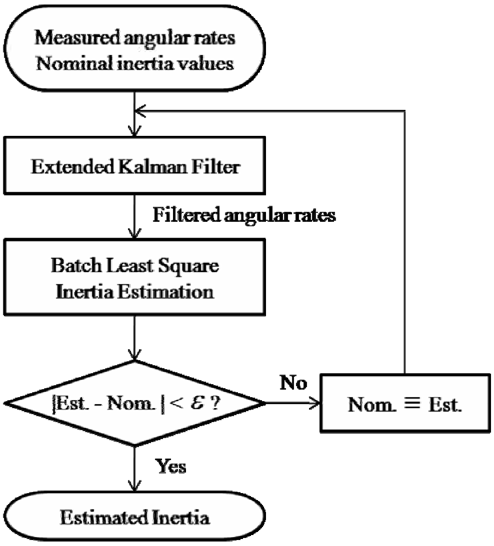

Fig. 6. Concept of the combined method.

shown in Table 2.

Here, the nominal inertia values, which have 10 percent error of the true values, are used for the EKF. The performance of the EKF with different a priori values is tested and the results are shown in Table 3.

It is quite apparent that the accuracy of estimation is improved using the better nominal value of the inertia properties as shown in Table 3. It is obvious that better estimation can be achieved when the estimated inertia is used as the nominal value. Iteration process is suggested to get more accurate properties and it is the key idea of this paper.

4. Combined estimation method

First, we reduce the noise with the EKF based on rough nominal inertia values and estimate the inertia properties using the batch method. Then, we substitute the estimated properties for initial values. Finally, we perform iteration until the estimated properties are converged. The concept of this method is shown in Fig. 6.

The results of the estimated inertia properties applying the above procedure are shown in Table 4.

The tendency of the convergence upon the number of iteration and the improvement of the estimated results are shown in Figs. 7 and 8, and the three-dimensional result is shown in Fig. 9.

In a simulation study, data were obtained every 0.25 seconds and 2400 data sets were used in our calculation. In addition, it takes less than 5 milliseconds to perform a 10 times iteration process on the following simulation environment:

Table 4. Performance of the Iteration method.

	Inertia (kg-m^2)		
	I_{xx}	I_{yy}	I_{zz}
True	14.2	17.3	20.3
Initial	12.78	15.57	18.27
Final	14.1522	17.0918	20.1256
Final error (%)	0.3367	1.2034	0.8593

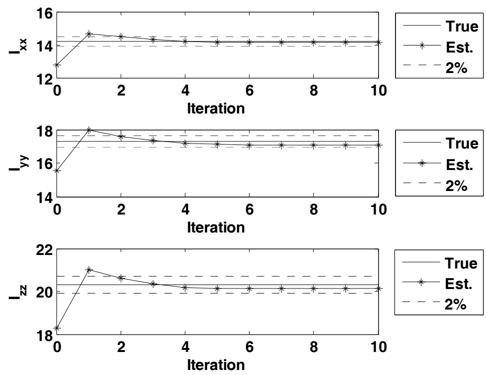

Fig. 7. Trajectory of the estimation.

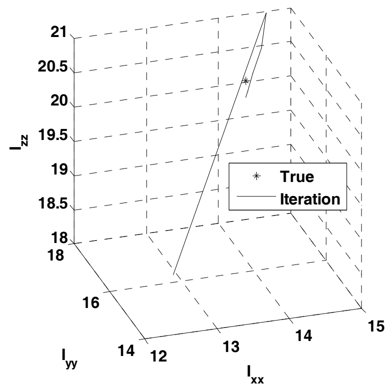

Fig. 8. Three dimensional trajectory of the estimation.

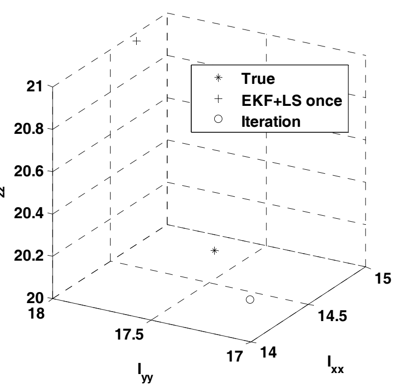

Fig. 9. Comparison of the estimation results.

Intel Pentium M Processor 740 (2M Cache, 1.73 GHz, 533 MHz FSB). It is possible to estimate inertia properties every 10 minutes on the assumption that the inertia properties are not changed rapidly.

In this paper, semi-real time possibility is shown on the assumption of gradual changes during 10 minutes.

5. Conclusions

A new method is suggested to improve the performance of the estimation of the inertia properties. The combined estimation method consists of two steps. First, the filtered data of angular rates are obtained using the EKF and they are used to the batch method for estimating inertia properties. Then, the iteration is performed with new initial values which are estimated in the previous step until they are converged. The improvement of estimation accuracy is achieved by using the suggested method.

Future work involves real-time application using the recursive least squares algorithm combined with the EKF.

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References

[1] S. Tanygin and T. Williams, Mass Property Estimation using Coasting Maneuvers, *Journal of Guidance, Control, and Dynamics*, 20 (4) (1997) 625-632.
[2] E. V. Bergmann, B. K. Walker and D. R. Levy, Mass Property Estimation for Control of Asymmetrical Satellites, *Journal of Guidance, Control, and Dynamics*, 10 (5) (1987) 483-491.
[3] J. A. Keim, A. B. Ackimese and J. F. Shields, Spacecraft Inertia Estimation via Constrained Least Squares, *Proc. of IEEE Aerospace Conference*, Big Sky, Montana (2006) 6-10.
[4] J. Palimaka and B. V. Burlton, Estimation of Spacecraft Mass Properties using Angular Rate Gyro Data, *Proc. of AIAA/AAS Astrodynamics Conference*, Hilton Head Island, SC (1992) 21-26.
[5] E. Wilson, C. Lages and R. Mah, On-line, Gyrobased, Mass Property Identification for thruster controlled Spacecraft using Recursive Least Squares, *Proc. of the 45th IEEE International Midwest Symposium on Circuits and Systems*, Tulsa, Oklahoma (2002) 334-337.
[6] M. L. Psiaki, F. Martel and P. K. Pal, Three-axis Attitude Determination via Kalman Filtering of Magnetometer Data, *Journal of Guidance, Control, and Dynamics*, 13 (3) (1990) 506-514.
[7] S. Herman, A Multi-mode Attitude Determination and Control System for SUNSAT, *PhD. Thesis, Department of Electronic Engineering*, University of Stellanbosch (1995).
[8] A. Kultu, C. Hacryev and O. Tekinalp, Attitude Determination and Rotational Motion Parameters Identification of a LEO Satellite through Magnetometer and Sun Sensor Data, *Proc. of 3rd International Conference on Recent Advances in Space Technologies*, Istanbul, Turkey (2007) 458-461.
[9] L. Alminde, Semi-Active Attitude Control and Off-line Attitude Determination for the SSETI-Express Student Micro-Satellite, *Proc. of 56th International Astronautical Congress*, Fukuoka, Japan (2005) IAC-05-E2.3.01.
[10] S. Yang, D. Cheon, S. Lee and H. Oh, On-orbit Estimation of Dynamic Properties for STSAT3, *International Conference on Control, Automation and Systems*, Coex, Seoul, Korea (2008) 459-462.
[11] P. C. Hughes, *Spacecraft Attitude Dynamics*, John Wiley & Sons, Inc. (1986).
[12] D. Simon, *Optimal State Estimation: Kalman, H_∞, and Nonlinear Approaches*, John Wiley & Sons, Inc. (2006) 79-81.
[13] The Math Works, *Signal Processing Toolbox 6: User's guide* (2006).
[14] M. Hattingh, A new Linear Phase, Robust, Nonlinear Smoothing Method for Stationary and Nonstationary Time or Space Series Data, *Proc. of Communication and Signal Processing*, Pretoria, South Africa (1988) 98-104.
[15] M. S. Grewal, A. P. Andrews, *Kalman Filtering: Theory and Practice using MATLAB*, 2nd Edition, John Wiley & Sons, Inc. (2001) 178-181.
[16] *STSAT-3 System Design Review*, KARI (2007).

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